# Quiz 2

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## Question 1

A) True, B) False, C) False, D) True, E) False

## Question 2

## A,B. Social Planner's problem and optimization

$$V(S,a,v) = \max_{C,E,X,S'} \{u(C) + \beta E[V(S',a',v')|a,v] + \lambda_c(a(1-E)S - C) + \lambda_s(-S' + S + X) + \lambda_x(vES - X)\}$$

$$\begin{aligned} u'(C) &= \lambda_c \\ \lambda_c a &= \lambda_x v \\ \lambda_x &= \lambda_s \\ \lambda_s &= \beta E[V'(S',a',v')|a,v] \\ V'(S,a,v) &= \lambda_s + \lambda_c a(1-E) + \lambda_x vE; \end{aligned}$$

It will be convenient to express the final two terms as

$$\lambda_c \frac{a}{v} = \beta E[\lambda_c'(\frac{a'}{v'} + a')|a,v]$$

## C. description of competitive equilibrium

- recursive household problem: household divides its hours to work and teaching

$$\begin{aligned} V(s,S,a,v) &= \max_{c,e,h,x}\{u(c) + \beta E[V(s',S',a',v')|a,v]\} \\ c + q(S,a,v)x &\leq r(S,a,v)sh + p(S,a,v)se \\ s' &= s + x \\ h + e &\leq 1 \\ S' &= G(S) \\ \log a' &= \log a + \epsilon_a \\ \log v' &= \log v + \epsilon_v \end{aligned}$$

- final goods firm's problem

$$\max_{S_f^d H^d} aS^d H^d - r(S,a,v)S^d H^d$$

- human capital investment firm's problem

$$\max_{S_i^d E^d} q(S,a,v)vS^d E^d - p(S,a,v)S^d E^d$$

- Market clearing: There are 4 goods in this economy

$$Sh(S,S,a,v) = S_f^d(S,a,v)H^d(S,a,v)$$

(1) $$Se(S,S,a,v) = S_i^d(S,a,v)E^d(S,a,v)$$

$$x(S,S,a,v) = vS^d(S,a,v)E^d(S,a,v)$$

(final good's market clearing is implied by other equilibrium conditions)

- Rational Expectation G(S)=S+x(S,S,a,v)

## D,E. optimality conditions for household and firm

- household
  Substituting c and e, we get

$$V(s,S,a,v) = \max_{h,s'}\{u(r(S,a,v)sh + p(S,a,v)s(1-h) - q(S,a,v)(s'-s)) + \beta E[V(s',S',a',v')|a,v]\}$$

  FOCs are

  (1) $h : r(S,a,v)s = p(S,a,v)s$

  (2) $s' : u'(c)q(S,a,v) = \beta E[V'(s',S',a',v')|a,v]$

  (3) $BS : V'(s,S,a,v) = u'(c)(r(S,a,v)h + p(S,a,v)(1-h) + q(S,a,v))$

  Combinig (3) and (4) gives

  (4) $x : u'(c)q(S,a,v) = \beta E[u'(c')(r(S',a',v')h' + p(S',a',v')(1-h') + q(S',a',v'))|a,v]$

- final goods firm: $0 \leq S_f^d(S,a,v)H^d(S,a,v) \leq \infty$ under price satisfying

  (5) $r(S,a,v) = a$

- investment firm: $0 \leq S_i^d(S,a,v)E^d(S,a,v) \leq \infty$ under price satisfying

  (6) $q(S,a,v)v = p(S,a,v)$

Market clearing (1) suggests (6),(7) to hold.
Substituting (6) into household FOCs (2) we get

(7) $p(S,a,v) = a$

thus by (7),

(8) $q(S,a,v) = \dfrac{a}{v}$

Using these and (5) and market clearing, c=C, we have

(9) $u'(C)\dfrac{a}{v} = \beta E[u'(C')(a' + \dfrac{a'}{v'})|a,v]$

We can see that this competitive equilibrium supports the planner's allocations by choosing:

$$u'(C) = \lambda_c$$

$$q = \frac{\lambda_x}{\lambda_c}$$

That is, u'(C) is the shadow price of consumption in the social planners problem and q is the ratio of the shadow price of investment to the shadow price of consumption.