Quiz 4

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Part I

A) False, B) False, C) True, D) False

Part II

1, cost minimization problem

For all t, cost minimization problem for firm i is, given $\{w_t, r_t\}$

$$
\begin{aligned}
& Min_{\{h_t,k_t\}} w_t h_t + r_t k_t \\
& s.t.\ y_t = k_t^{\alpha}(A_t h_t)^{1-\alpha}
\end{aligned}
$$

2, derivation of the marginal cost

The FOCs for the above problem are, using Lagrange multiplier λ,

$$
\begin{aligned}
h : w &= (1-\alpha)k^{\alpha}h^{-\alpha}A^{1-\alpha}\lambda \\
k : r &= \alpha k^{\alpha-1}h^{1-\alpha}A^{1-\alpha}\lambda,
\end{aligned}
$$

from which we get

$$k = \frac{w}{r}\frac{\alpha}{1-\alpha}h.$$

Using this and production function, k and h are solved as

$$
\begin{aligned}
h &= y(\frac{r}{w}\frac{1-\alpha}{\alpha})^{\alpha}A^{\alpha-1} \\
k &= y(\frac{r}{w}\frac{1-\alpha}{\alpha})^{\alpha-1}A^{\alpha-1}.
\end{aligned}
$$

Substituting these into the objective function, cost function can be written as

$$C(y,w,r,A) = \left[w(\frac{r}{w}\frac{1-\alpha}{\alpha})^{\alpha}A^{\alpha-1} + r(\frac{r}{w}\frac{1-\alpha}{\alpha})^{\alpha-1}A^{\alpha-1}\right]y,$$

hence, marginal cost is

$$
\begin{aligned}
\frac{\partial C(y,w,r,A)}{\partial y} &= w(\frac{r}{w}\frac{1-\alpha}{\alpha})^{\alpha}A^{\alpha-1} + r(\frac{r}{w}\frac{1-\alpha}{\alpha})^{\alpha-1}A^{\alpha-1} \\
&= \frac{r^{\alpha}w^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A^{1-\alpha}} \\
&\equiv \chi(w,r,A).
\end{aligned}
$$

Note that we can now write cost function as

$$C(y, w, r, A) = \chi(w, r, A)y.$$

3, dynamic price setting problem

The dynamic price setting problem for firm i is, [1]

$$Max_{\{p_t(i)\}_t} E_0 \sum_{t=0}^{\infty} \beta^t (p_t(i) - \chi(w_t, r_t, A_t) - \frac{\gamma}{2}(\frac{p_t(i)}{p_{t-1}(i)} - 1)^2) y_t(i)$$

$$s.t.\ y_t(i) = (\frac{p_t(i)}{P_t})^{-\theta} y_t$$

4, FOC and its interpretation

Rearranging the terms, we get the following FOC

$$(1-\theta)p_t(i) + \chi(w_t, r_t, A_t)\theta + \frac{\gamma}{2}(\frac{p_t(i)}{p_{t-1}(i)} - 1)^2\theta - \gamma(\frac{p_t(i)}{p_{t-1}(i)} - 1)\frac{p_t(i)}{p_{t-1}(i)} + \beta\gamma(\frac{p_{t+1}(i)}{p_t(i)} - 1)\frac{p_{t+1}(i)}{p_t(i)} = 0.$$

When there is no adjustment cost, the terms except for the first two disappear and the price is determined as the constant mark up to the marginal cost

$$p_t(i) = \frac{\theta}{\theta - 1}\chi(w_t, r_t, A_t).$$

When there is adjustment cost as above, the mark up may no longer be constant, depending on the inflation rate and sequence of maginal costs. When price is constant, it coincides with FOC of no adjustment cost.

[1] Here, we assume technology shock, A_t, to be stochastic, so that firm i maximizes its expected profit