Quiz 3

Winter term 2009

Draft

Part 1

(1) False (2) True (3) False (4) True (5) False

Part 2

(1)

The Social planner will choose to maximize a weighted sum of individual utilities subject to the aggregate resource constraint. The aggregate resource constraint for this economy is given by:

$$N(y_1 + y_2) = N(c_1 + c_2)$$

Note next that the law of motions for endowments implies that the aggregate endowment is constant in each period:

$$y(0) \equiv y_1(0) + y_2(0) = y_1(1) + y_2(1) = \ldots = y_1(t) + y_2(t)$$

We will assume that the social planner treats all individuals of a given type the same way.

The Recursive Social Planner's problem is

$$V(y_1, y_2) = \max_{c_1, c_2} \left\{ \varphi N\left(c_1 - \frac{c_1^2}{4}\right) + (1-\varphi)N\left(c_2 - \frac{c_2^2}{4}\right) + \beta E\left[V(y_1', y_2' \mid y_1, y_2)\right] \right\}$$

$$\begin{aligned} st \quad & N(c_1 + c_2) = N(y_1 + y_2) \\ & y_1' = y_1 + \sigma\varepsilon' \\ & y_2' = y_{21} - \sigma\varepsilon' \end{aligned}$$

Using the above fact about the aggregate endowment we can simplify the problem to:

$$V(y) = \max_{c_1, c_2} \left\{ \varphi N\left(c_1 - \frac{c_1^2}{4}\right) + (1-\varphi)N\left(c_2 - \frac{c_2^2}{4}\right) + \beta\left[V(y)\right] \right\}$$

$$st \quad N(c_1 + c_2) = Ny$$

The F.O.C.s are (let λ be the Lagrange multiplier on the resource constraint)

$$c_1 : \varphi N(1 - \frac{c_1}{2}) - \lambda N = 0$$

$$c_2 : (1-\varphi)N(1 - \frac{c_2}{2}) - \lambda N = 0$$

Combining the two expressions we get:

$$(1 - \frac{c_1}{2}) = \frac{(1-\varphi)}{\varphi}(1 - \frac{c_2}{2})$$

which links consumption of each type to the social planner's weight.

The case of equal weights is interesting because these allocations can be supported with out a tax transfer scheme. Under equal weights we have:

$y_1 = y_2 = \frac{1}{10}$, $c_1 = c_2 = \frac{1}{10}$.

Since this utility function does not satisfy the Inada condition, we need to consider the corner solution. Without loss of generality, we set $(c_{1,t}, c_{2.t}) = (2y.0)$ for all t. Then

$$2\left(\frac{1}{10} - \frac{1}{4}\left(\frac{1}{10}\right)^2\right) > \frac{1}{5} - \frac{1}{4}\left(\frac{1}{5}\right)^2$$

Hence, $c_1 = c_2 = \frac{1}{10}$ is actually the optimal allocations for the Pareto problem under the assumption that $\varphi = 1/2$.

(2)

Use guess and verify. Notice first, that under this guess the resource constraint is satisfied since each individual consumptions his/her endowment in each period.

To derive the equilibrium price on the risk-free bond consider a typical individual's optimization problem:

$$V(y_i) = \max_{c_i}\left\{\left(c_i - \frac{c_1^2}{4}\right) + \beta E\left[V(y_i') \mid y_i\right]\right\}$$

$$st \quad c_i + R^{-1}b_i' = y_i + b$$

and the law of motion for the endowment.

The FONC's for the individual's problem imply:

$$\beta R_t E_t\left[\frac{(1 - \frac{c_i'}{2})}{(1 - \frac{c_i}{2})}\right] = 1$$

Next use the guess:

$$\beta R_t E_t\left[\frac{(1 - \frac{y_i'}{2})}{(1 - \frac{y_i}{2})}\right] = 1$$

or

$$\beta R_t \left[\frac{(1-\frac{E_t y_i^{'}}{2})}{(1-\frac{y_i}{2})} \right] = 1$$

or,

$$\beta R_t \left[\frac{(1-\frac{y_i}{2})}{(1-\frac{y_i}{2})} \right] = 1$$

or,

$$\beta R_t = 1$$

Note that this restriction on the risk free rate is identical for each type of household so there is agreement on the price of the risk free bond by each type of agent and thus the bond market clears.

Finally, note from the household budget constraint that under our guess holdings of bonds evolve in the following way:

$$b_i(1) = \frac{1}{\beta} b_i(0)$$

$$b_i(2) = \frac{1}{\beta} b_i(1) = \frac{1}{\beta^2} b_i(0)$$

…

$$b_i(t) = \frac{1}{\beta^t} b_i(0)$$