The Basic New Keynesian Model

by

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Motivation and Outline

Evidence on Money, Output, and Prices:

- Short Run Effects of Monetary Policy Shocks

  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

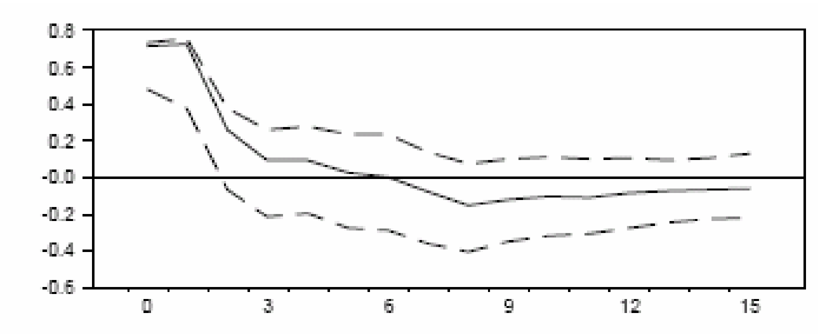
- Micro Evidence on Price-setting Behavior: significant price and wage rigidities

Failure of Classical Monetary Models

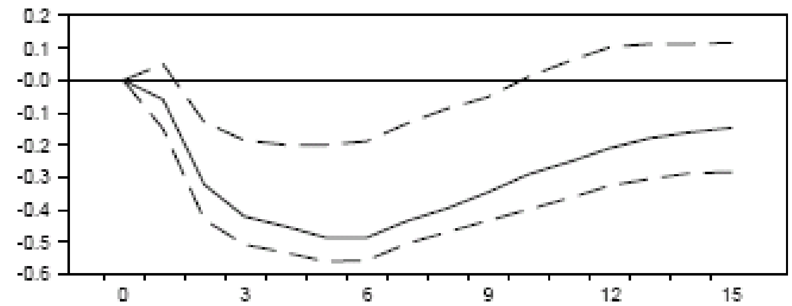
A Baseline Model with Nominal Rigidities

- monopolistic competition
- sticky prices (staggered price setting)
- competitive labor markets, closed economy, no capital accumulation

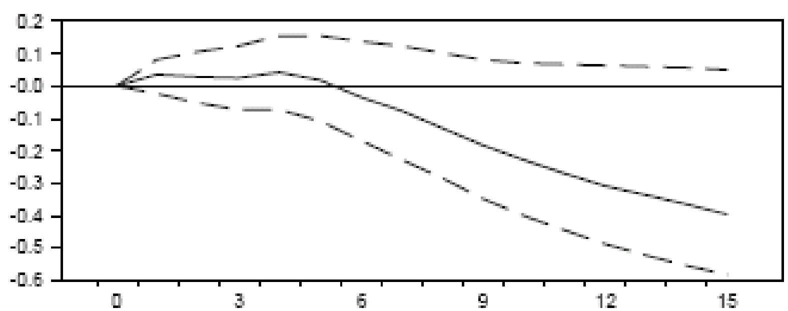
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



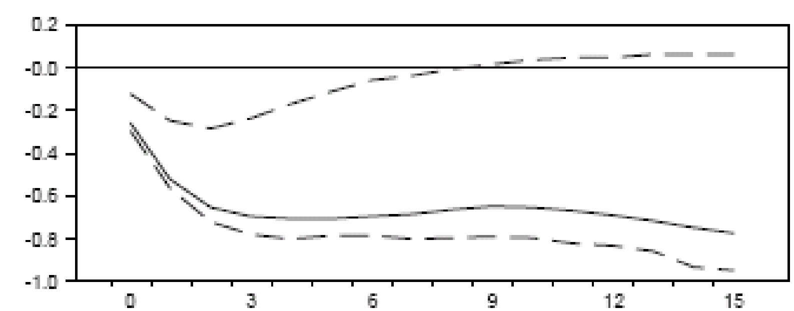
Federal funds rate



GDP



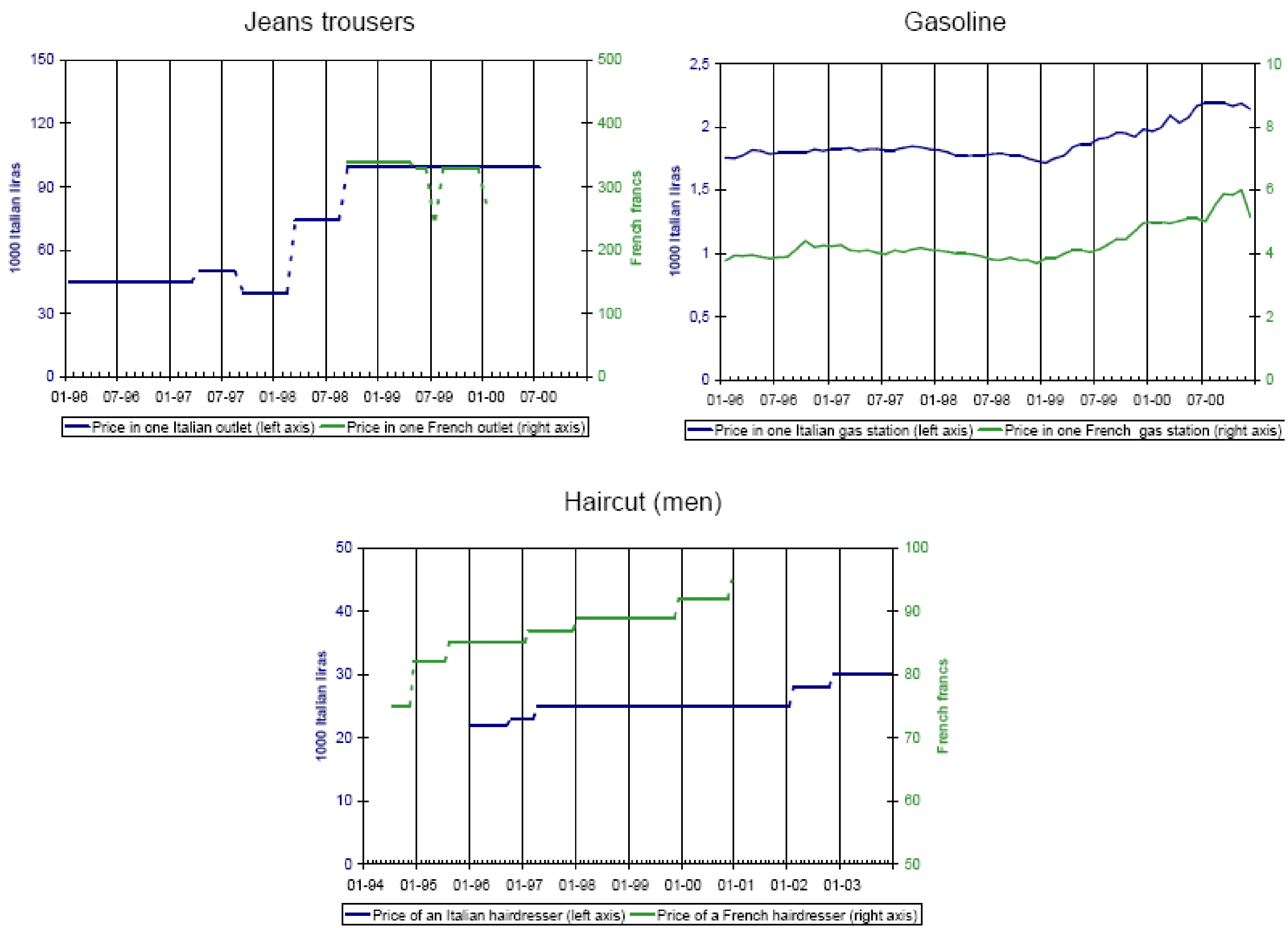
GDP deflator



M2

Source: Christiano, Eichenbaum and Evans (1999)

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al*. (2004) and Veronese *et al*. (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. WP 05

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, N_t\right)$$

where

$$C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \, di\right]^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) \, C_t(i) \, di + Q_t \, B_t \leq B_{t-1} + W_t \, N_t - T_t$$

for $t = 0, 1, 2, \ldots$ plus solvency constraint.

Optimality conditions

1. Optimal allocation of expenditures

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \; C_t$$

implying

$$\int_0^1 P_t(i) \; C_t(i) \; di = P_t \; C_t$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} \; di \right]^{\frac{1}{1-\epsilon}}$$

2. Other optimality conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta \; E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \; \frac{P_t}{P_{t+1}} \right\}$$

Specification of utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

implied log-linear optimality conditions (aggregate variables)

$$w_t - p_t = \sigma \ c_t + \varphi \ n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho\right)$$

where $i_t \equiv -\log Q_t$ is the *nominal interest rate* and $\rho \equiv -\log \beta$ is the *discount rate.*

Ad-hoc money demand

$$m_t - p_t = y_t - \eta \ i_t$$

Firms

- Continuum of firms, indexed by $i \in [0,1]$
- Each firm produces a differentiated good
- Identical technology
$$Y_t(i) = A_t \; N_t(i)^{1-\alpha}$$
- Probability of resetting price in any given period: $1-\theta$, independent across firms (Calvo (1983)).
- $\theta \in [0,1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$

Aggregate Price Dynamics

$$P_t = \left[\theta \ (P_{t-1})^{1-\epsilon} + (1-\theta) \ (P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Dividing by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \ \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

Log-linearization around zero inflation steady state

$$\pi_t = (1-\theta) \ (p_t^* - p_{t-1}) \tag{1}$$

or, equivalently

$$p_t = \theta \ p_{t-1} + (1-\theta) \ p_t^*$$

Optimal Price Setting

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \; \left(P_t^* \; Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to

$$Y_{t+k|t} = (P_t^* / P_{t+k})^{-\epsilon} \; C_{t+k}$$

for $k = 0, 1, 2, \ldots$where

$$Q_{t,t+k} \equiv \beta^k \; \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$$

Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \; Y_{t+k|t} \; \left(P_t^* - \mathcal{M} \; \psi_{t+k|t} \right) \right\} = 0$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$

Equivalently,

$$\sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} \ Y_{t+k|t} \ \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} \ MC_{t+k|t} \ \Pi_{t-1,t+k} \right) \right\} = 0$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ and $\Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1}$

Perfect Foresight, Zero Inflation Steady State:

$$\frac{P_t^*}{P_{t-1}} = 1 \quad ; \quad \Pi_{t-1,t+k} = 1 \quad ; \quad Y_{t+k|t} = Y \quad ; \quad Q_{t,t+k} = \beta^k \quad ; \quad MC = \frac{1}{\mathcal{M}}$$

Log-linearization around zero inflation steady state:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}\}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$.

Equivalently,

$$p_t^* = \mu + (1-\beta\theta)\;\sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k|t} + p_{t+k}\}$$

where $\mu \equiv \log \frac{\epsilon}{\epsilon - 1}$.

Flexible prices ($\theta = 0$):

$$p_t^* = \mu + mc_t + p_t$$

$\Longrightarrow \qquad mc_t = -\mu$ (symmetric equilibrium)

Particular Case: $\alpha = 0$ (constant returns)

$$\Longrightarrow \; MC_{t+k|t} = MC_{t+k}$$

Rewriting the optimal price setting rule in recursive form:

$$p_t^* = \beta\theta \; E_t\{p_{t+1}^*\} + (1-\beta\theta) \; \widehat{mc}_t + (1-\beta\theta) p_t \tag{2}$$

Combining (1) and (2):

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Generalization to $\alpha \in (0,1)$ (decreasing returns)

Define

$$
\begin{aligned}
mc_t &\equiv (w_t - p_t) - mpn_t \\
&\equiv (w_t - p_t) - \frac{1}{1-\alpha}\,(a_t - \alpha y_t) - \log(1-\alpha)
\end{aligned}
$$

Using $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}\,(a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha)$,

$$
\begin{aligned}
mc_{t+k|t} &= mc_{t+k} + \frac{\alpha}{1-\alpha}\,(y_{t+k|t} - y_{t+k}) \\
&= mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}\,(p_t^* - p_{t+k}) \qquad (3)
\end{aligned}
$$

Implied inflation dynamics

$$
\pi_t = \beta\, E_t\{\pi_{t+1}\} + \lambda\, \widehat{mc}_t \qquad (4)
$$

where

$$
\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\,\frac{1-\alpha}{1-\alpha+\alpha\epsilon}
$$

Equilibrium

Goods markets clearing

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all t.

Letting $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}}$,

$$Y_t = C_t$$

for all t. Combined with the consumer's Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) \tag{5}$$

Labor market clearing

$$
\begin{aligned}
N_t &= \int_0^1 N_t(i)\; di \\
&= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} \; di \\
&= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} \; di
\end{aligned}
$$

Taking logs,

$$(1-\alpha)\; n_t = y_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log \int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}}\; di$ (second order).

Up to a first order approximation:

$$y_t = a_t + (1-\alpha)\; n_t$$

Marginal Cost and Output

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\sigma\ y_t + \varphi\ n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\ y_t - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \end{aligned} \tag{6}$$

Under *flexible prices*

$$mc = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\ y_t^n - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \tag{7}$$

$$\Longrightarrow\ y_t^n = -\delta_y + \psi_{ya}\ a_t$$

where $\delta_y \equiv \frac{(\mu - \log(1-\alpha))(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$.

$$\Longrightarrow\ \widehat{mc}_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\ (y_t - y_t^n) \tag{8}$$

where $y_t - y_t^n \equiv \widetilde{y}_t$ is the *output gap*

New Keynesian Phillips Curve

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t \tag{9}$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

Dynamic IS equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) \tag{10}$$

where r_t^n is the *natural rate of interest*, given by

$$\begin{aligned} r_t^n &\equiv \rho + \sigma \; E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma\psi_{ya} \; E_t\{\Delta a_{t+1}\} \end{aligned}$$

Missing block: description of monetary policy (determination of i_t).

Equilibrium under a Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t + v_t \tag{11}$$

where v_t is exogenous (possibly stochastic) with zero mean.

Equilibrium Dynamics: combining (9), (10), and (11)

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \, (\widehat{r}_t^n - v_t) \tag{12}$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$

Uniqueness $\iff \mathbf{A}_T$ has both eigenvalues within the unit circle

Given $\phi_\pi \geq 0$ and $\phi_y \geq 0$, (Bullard and Mitra (2002)):

$$\kappa\,(\phi_\pi - 1) + (1-\beta)\,\phi_y > 0$$

is necessary and sufficient.

Effects of a Monetary Policy Shock

Set $\widehat{r}_t^n = 0$ (no real shocks).

Let v_t follow an AR(1) process

$$v_t = \rho_v \; v_{t-1} + \varepsilon_t^v$$

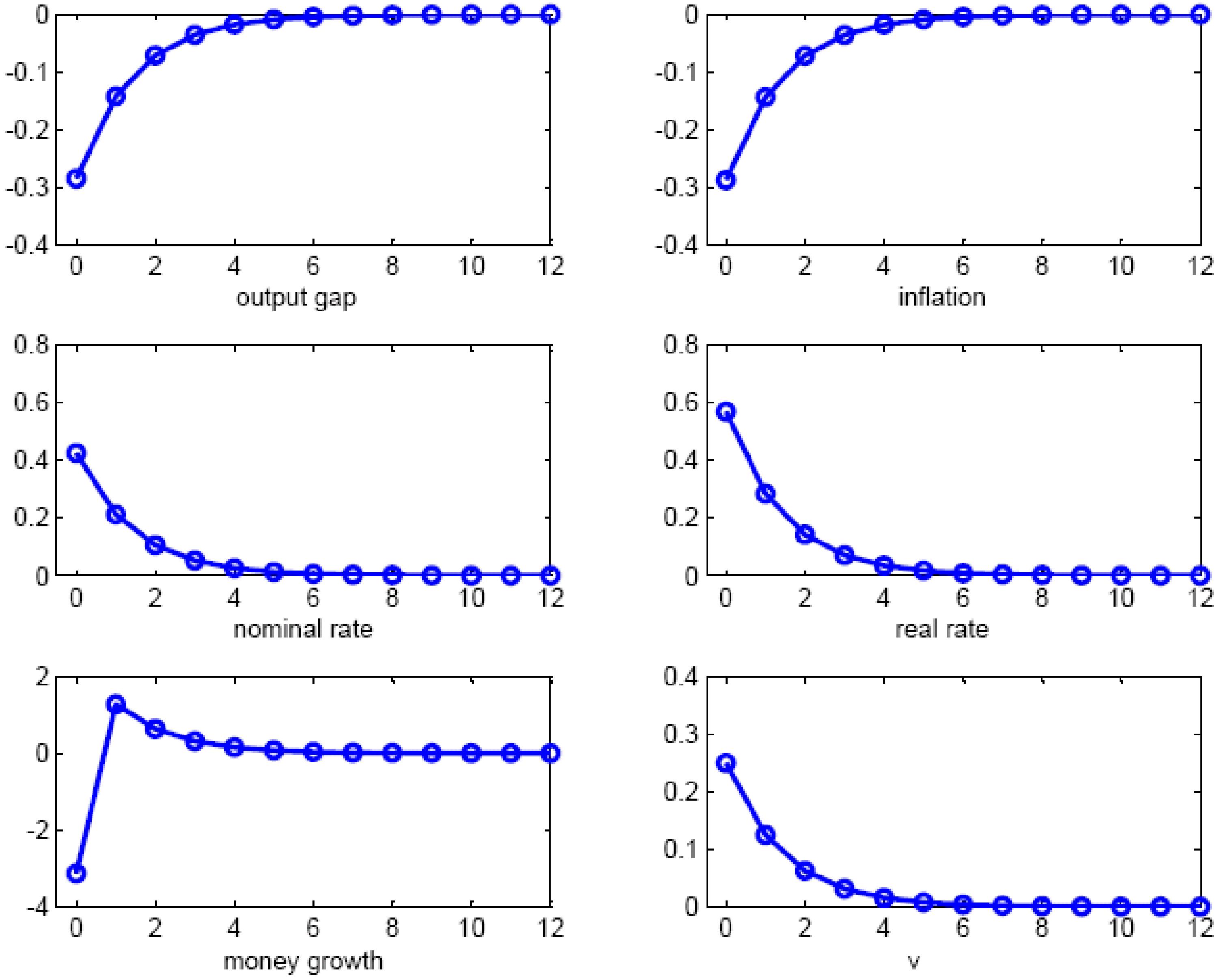
Calibration:

$$\rho_v = 0.5,\ \phi_\pi = 1.5,\ \phi_y = 0.5/4,\ \beta = 0.99,\ \sigma = \varphi = 1,\ \theta = 2/3,\ \eta = 4.$$

Dynamic effects of an exogenous increase in the nominal rate (Figure 1).

Exercise: analytical solution

Figure 3.1: Effects of a Monetary Policy Shock (Interest Rate Rule))



Effects of a Technology Shock

Set $v_t = 0$ (no monetary shocks).

Technology process:

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a.$$

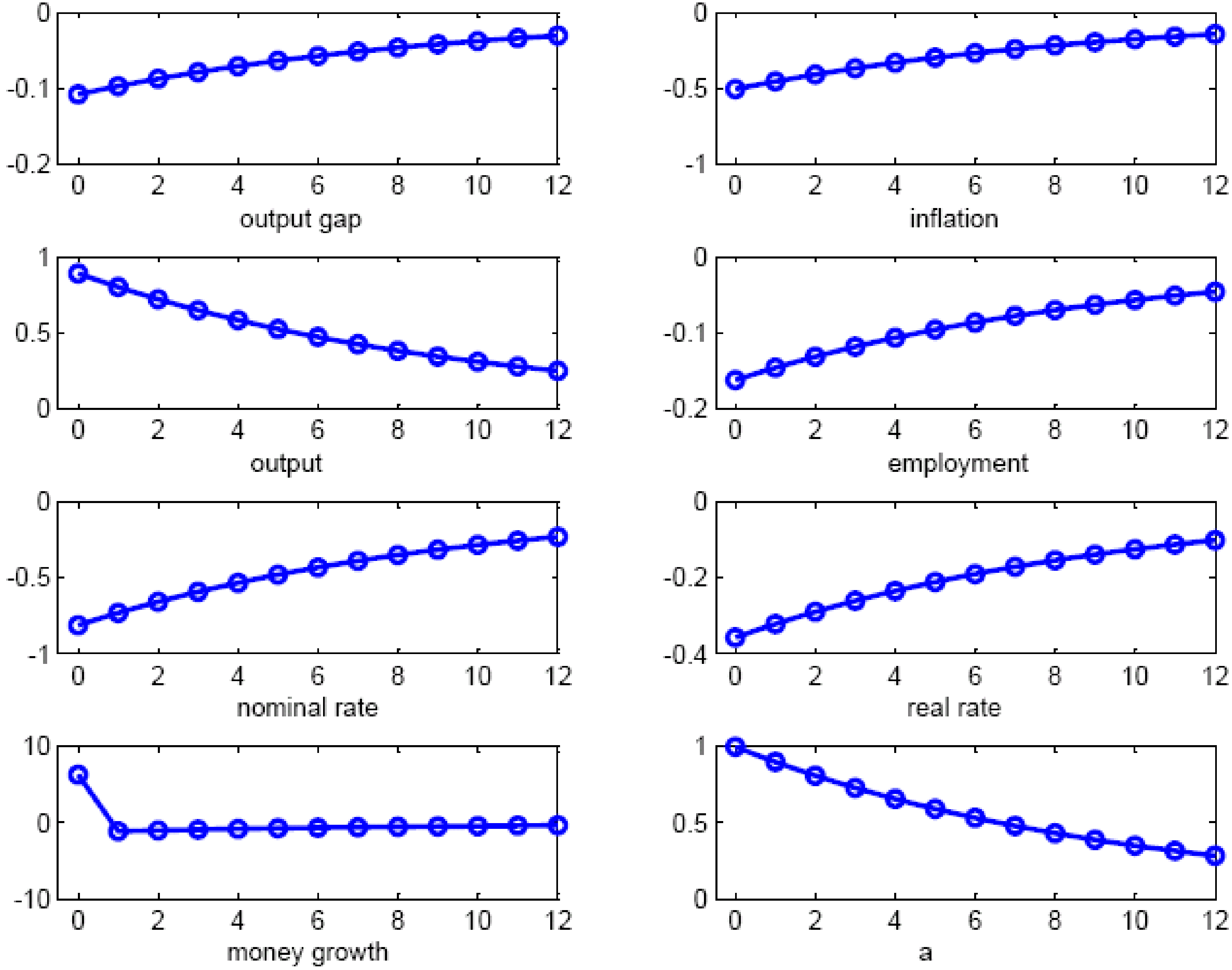
Implied natural rate:

$$\widehat{r}_t^n = -\sigma \psi_{ya}(1 - \rho_a) \; a_t$$

Dynamic effects of a technology shock ($\rho_a = 0.9$) (Figure 2)

Exercise: AR(1) process for Δa_t

Figure 3.2: Effects of a Technology Shock (Interest Rate Rule)



Equilibrium under an Exogenous Money Growth Process

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{13}$$

Money market clearing

$$\begin{aligned} \widehat{l_t} &= \widehat{y}_t - \eta \, \widehat{i}_t && (14) \\ &= \widetilde{y}_t + \widehat{y}_t^n - \eta \, \widehat{i}_t && (15) \end{aligned}$$

where $l_t \equiv m_t - p_t$ denotes (log) real money balances.

Substituting (14) into (10):

$$(1 + \sigma\eta) \;\; \widetilde{y}_t = \sigma\eta \; E_t\{\widetilde{y}_{t+1}\} + \widehat{l_t} + \eta \; E_t\{\pi_{t+1}\} + \eta \; \widehat{r}_t^n - \widehat{y}_t^n \tag{16}$$

Furthermore, we have

$$\widehat{l}_{t-1} = \widehat{l_t} + \pi_t - \Delta m_t \tag{17}$$

Equilibrium dynamics

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_{t-1} \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta m_t \end{bmatrix} \qquad (18)$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Uniqueness $\Longleftrightarrow$ $\mathbf{A_M} \equiv \mathbf{A_{M,0}^{-1}A_{M,1}}$ has two eigenvalues inside and one outside the unit circle.

Effects of a Monetary Policy Shock

Set $\widehat{r}_t^n = y_t^n = 0$ (no real shocks).

Money growth process

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{19}$$

where $\rho_m \in [0, 1)$

Figure 3 (based on $\rho_m = 0.5$)

Effects of a Technology Shock

Set $\Delta m_t = 0$ (no monetary shocks).

Technology process:

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a.$$

Figure 4 (based on $\rho_a = 0.9$).

Empirical Evidence

Figure 3.3: Effects of a Monetary Policy Shock (Money Growth Rule)

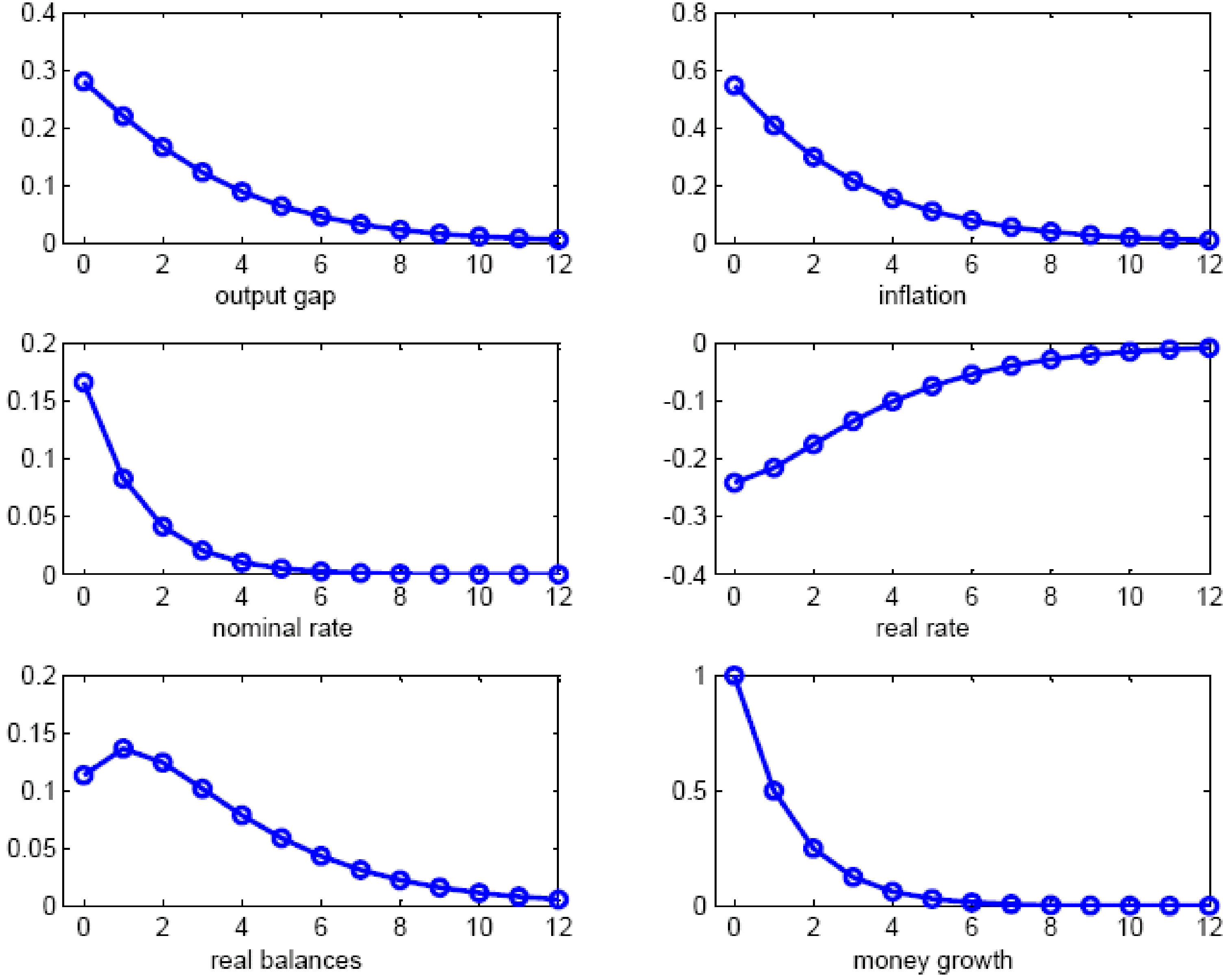


Figure 3.4: Effects of a Technology Shock (Money Growth Rule)

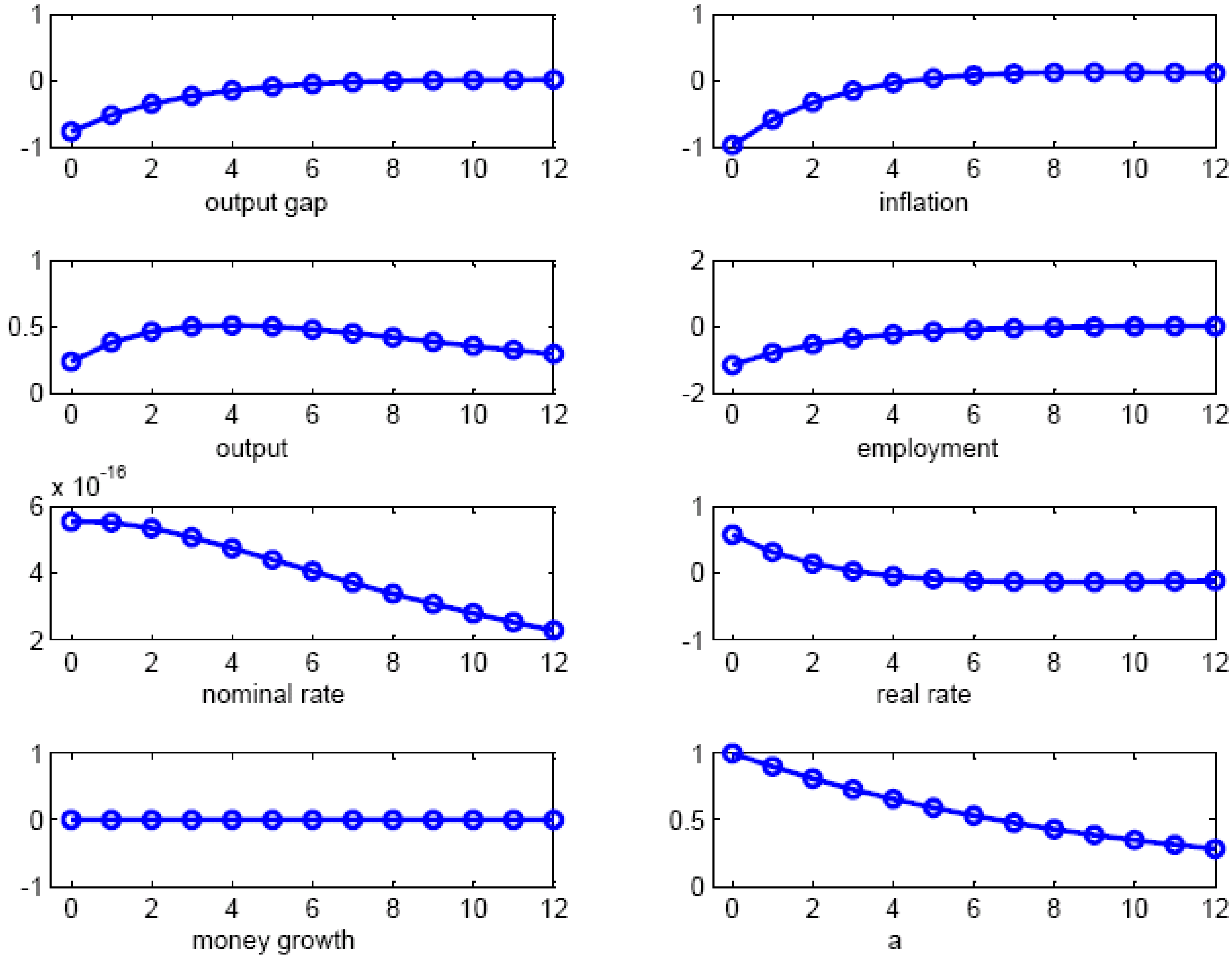


Figure 3.5: Estimated Effects of a Permanent Technology Shock

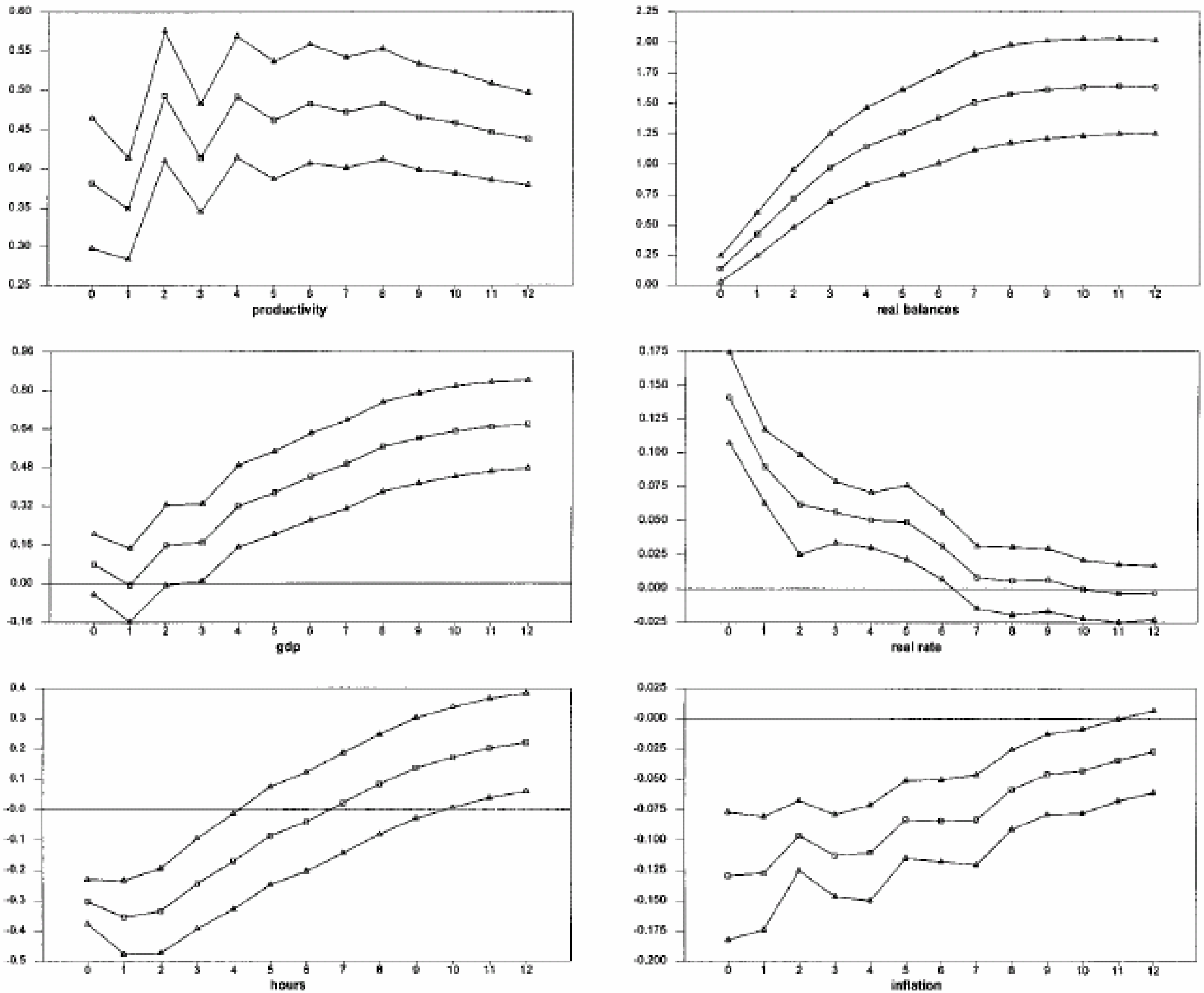


FIGURE 4. ESTIMATED IMPULSE RESPONSES FROM A FIVE-VARIABLE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS
(POINT ESTIMATES AND ±2 STANDARD ERROR CONFIDENCE INTERVALS)

Source: Galí (1999)

Technical Appendix

Optimal Allocation of Consumption Expenditures

Maximization of C_t for any *given* expenditure level $\int_0^1 P_t(i)\ C_t(i)\ di \equiv Z_t$ can be formalized by means of the Lagrangean

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\ di\right]^{\frac{\epsilon}{\epsilon-1}} - \lambda\ \left(\int_0^1 P_t(i)\ C_t(i)\ di - Z_t\right)$$

The associated first order conditions are:

$$C_t(i)^{-\frac{1}{\epsilon}}\ C_t^{\frac{1}{\epsilon}} = \lambda\ P_t(i)$$

for all $i \in [0,1]$. Thus, for any two goods (i,j) we have:

$$C_t(i) = C_t(j)\ \left(\frac{P_t(i)}{P_t(j)}\right)^{-\epsilon}$$

which can be plugged into the expression for consumption expenditures to yield

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\ \frac{Z_t}{P_t}$$

for all $i \in [0,1]$. The latter condition can then be substituted into the definition of C_t, yielding

$$\int_0^1 P_t(i)\ C_t(i)\ di = P_t\ C_t$$

Combining the two previous equations we obtain the demand schedule:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\ C_t$$

Log-Linearized Euler Equation

We can rewrite the Euler equation as

$$1 = E_t\{\exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho)\} \tag{20}$$

In a perfect foresight steady state with constant inflation π and constant growth γ we must have:

$$i = \rho + \sigma\gamma + \pi$$

with the steady state real rate being given by

$$\begin{aligned} r &\equiv i - \pi \\ &= \rho + \sigma\gamma \end{aligned}$$

A first order Taylor expansion of $\exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho)$ around that steady state yields:

$$\begin{aligned} \exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho) &\simeq 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) \\ &= 1 + i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho \end{aligned}$$

which can be used in (20) to obtain, after some rearrangement of terms, the log-linearized Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Aggregate Price Level Dynamics

Let $S(t) \subset [0,1]$ denote the set of firms which do not re-optimize their posted price in period t. The aggregate price level evolves according to

$$\begin{aligned} P_t &= \left[\int_{S(t)} P_{t-1}(i)^{1-\epsilon}\, di + (1-\theta)\,(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\ &= \left[\theta\,(P_{t-1})^{1-\epsilon} + (1-\theta)\,(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \end{aligned}$$

where the second equality follows from the fact that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, with total mass reduced to θ.

Equivalently, dividing both sides by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta \ + (1-\theta) \ \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{21}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $P_t^* = P_{t-1}$.

Log-linearization around a zero inflation ($\Pi = 1$) steady state implies:

$$\pi_t = (1-\theta) \ (p_t^* - p_{t-1}) \tag{22}$$

Price Dispersion

From the definition of the price index:

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} \ di \\ &= \int_0^1 \exp\{(1-\epsilon)(p_t(i) - p_t)\} \ di \\ &\simeq 1 + (1-\epsilon) \int_0^1 (p_t(i) - p_t) \ di + \frac{(1-\epsilon)^2}{2} \int_0^1 (p_t(i) - p_t)^2 \ di \end{aligned}$$

thus implying the second order approximation

$$p_t \simeq E_i\{p_t(i)\} + \frac{(1-\epsilon)}{2} \int_0^1 (p_t(i) - p_t)^2 \ di$$

where $E_i\{p_t(i)\} \equiv \int_0^1 p_t(i)\ di$ is the cross-sectional mean of (log) prices.
In addition,

$$\begin{aligned}
\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di &= \int_0^1 \exp\left\{-\frac{\epsilon}{1-\alpha}\ (p_t(i)-p_t)\right\}\ di \\
&\simeq 1-\frac{\epsilon}{1-\alpha}\int_0^1 (p_t(i)-p_t)\ di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2\ di \\
&\simeq 1+\frac{1}{2}\frac{\epsilon(1-\epsilon)}{1-\alpha}\int_0^1 (p_t(i)-p_t)^2\ di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2\ di \\
&= 1+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\int_0^1 (p_t(i)-p_t)^2\ di \\
&\simeq 1+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\ var_i\{p_t(i)\} > 1
\end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, and where the last equality follows from the observation that, up to second order,

$$\begin{aligned}
\int_0^1 (p_t(i)-p_t)^2\ di &\simeq \int_0^1 (p_t(i)-E_i\{p_t(i)\})^2\ di \\
&\equiv var_i\{p_t(i)\}
\end{aligned}$$

Finally, using the definition of d_t we obtain

$$d_t \equiv (1-\alpha)\log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \simeq \frac{1}{2}\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\}$$